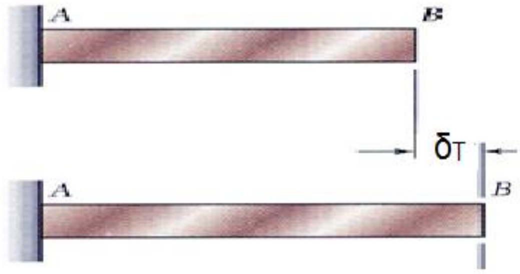
Thermal Stresses

Temperature changes cause the body to expand or contract. Mechanical stress induced in a body when some or all of its parts are not free to expand or contract in response to changes in temperature. In most continuous bodies, thermal expansion or contraction cannot occur freely in all directions because of geometry, external constraints, or the existence of temperature gradients, and so stresses are produced. Such stresses caused by a temperature change are known as thermal stresses.

The amount δ_T which gives the total thermal deflection (deformation due to temperature changes) is given by:



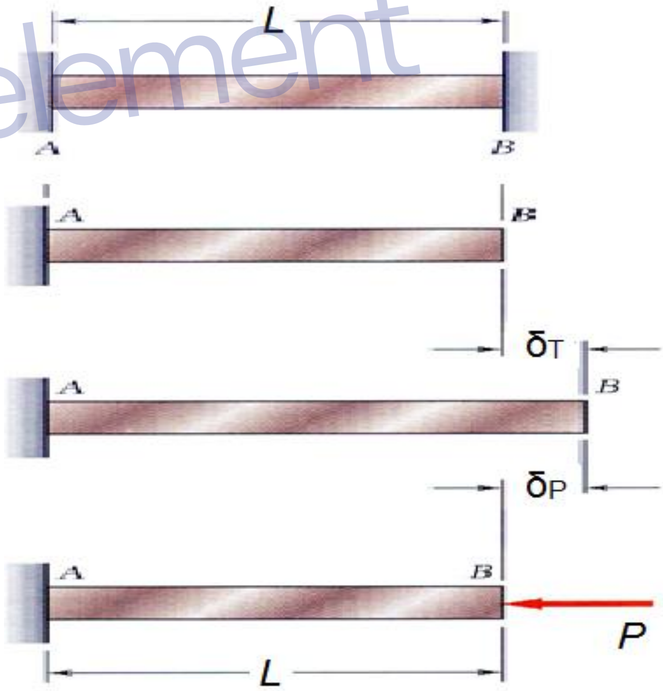
$$\delta_T = \alpha\, L\, (Tf - Ti) = \alpha\, L\, \Delta T$$

Where: δ_T: Deformation due to temperature changes

α: The coefficient of thermal expansion in m/m C°,

L: The length in meter,

T_i and T_f : are the initial and final temperatures, respectively in °C.



Deformation due to equivalent axial stress:

$$\delta_T = \delta_P$$

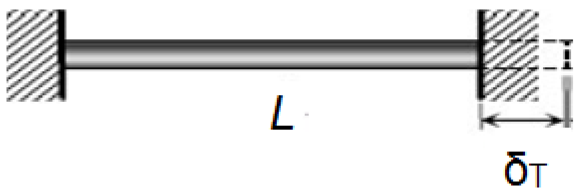
$$\delta_P = \frac{P\,.\,L}{A\;\;E} = \frac{\sigma\,.\,L}{E}$$

$$\alpha\,.\,L\;\;\Delta T = \frac{\sigma\,.\,L}{E}$$

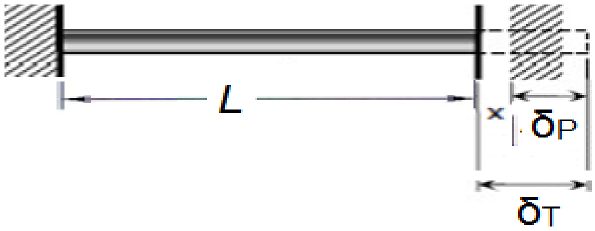
Which means : $\sigma = \alpha\, \Delta T\, E$

Where σ: is the thermal stress in MPa, E is the modulus of elasticity of the rod in MPa.

δ_P: Compression deformation



If the wall yields a distance of (x) as shown, the following calculations will be made:



That means: $\delta_T = x + \delta_P$

Or that: $\alpha \cdot L \, \Delta T = x + \sigma L/E$

Where σ represents the thermal stress

Keep in mind that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Example

A 5 m aluminum flagpole is installed at 20°C. Overnight, the temperature drops to -5°C. How much does the height change, in millimeters? What is the final height of the flagpole, in meters?

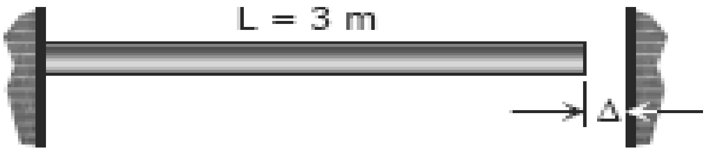
Solution First, calculate the change in length using $\delta = \alpha L(\Delta T)$. From the Appendix, the thermal expansion coefficient for aluminum is $\alpha_{Aluminum} = 23 \times 10^{-6} °C^{-1}$. Next, calculate the final length by adding the change in length to the original length.

Change in length $\delta = \alpha L(\Delta T) = \dfrac{23 \times 10^{-6}}{°C} \dfrac{5\,\text{m}(-5°C - 20°C)}{} \Big| \dfrac{10^3\,\text{mm}}{\text{m}} = -2.88\,\text{mm}$. The negative sign indicates the flagpole is getting shorter.

Final length $L_f = L + \delta = 5\,\text{m} - \dfrac{2.88\,\text{mm}}{} \Big| \dfrac{\text{m}}{10^3\,\text{mm}} = 4.997\,\text{m}$

Example:

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in the figure at a temperature of -20°C, the gap Δ = 2.5 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use $\alpha = 18.0 \times 10^{-6}$ m/(m·°C) and E = 80 GPa.



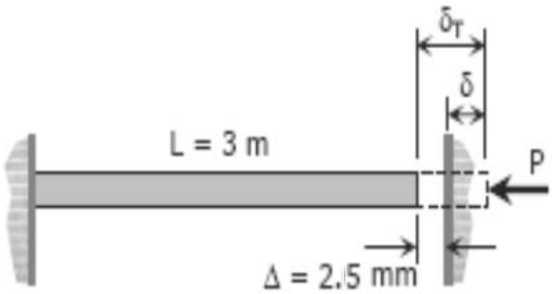
Solution:

$$\delta_T = \delta + \Delta$$

$$\alpha L(\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

$$\Delta T = 70.6°C$$



$$\Delta T = T2 - T1$$

$$T2 = \Delta T + T1$$
$$= 70.6 - 20$$
$$= 50.6°C$$

Ex:

Two immovable concrete blocks are connected by a steel wire. At 72°F there is no stress in the wire. If the wire cools from 72°F to 55°F. what is the stress in the wire?

Solution Find these steel properties in the Appendix: $\alpha = 6.5 \times 10^{-6}\,°F^{-1}$ and $E = 30 \times 10^{6}$ psi .

Thermal stress $\sigma = -\alpha E(\Delta T) = \dfrac{-6.5 \times 10^{-6}}{°F} \dfrac{30 \times 10^{6}\,\text{lb.}}{\text{in.}^2} (55\,°F - 72\,°F) = 3{,}315$ psi

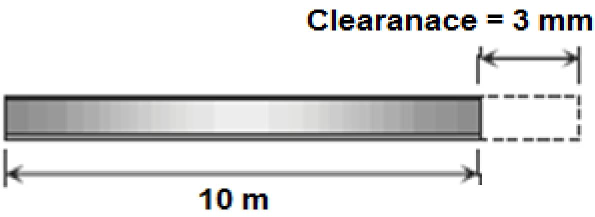
The positive sign indicates the wire is under a tensile stress. The wire cooled and wanted to shrink, but the concrete blocks prevented it from shrinking, leaving the wire in tension.

Ex:

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. Assuming α = 11.7 µm/(m·°C) and E = 200 GPa.

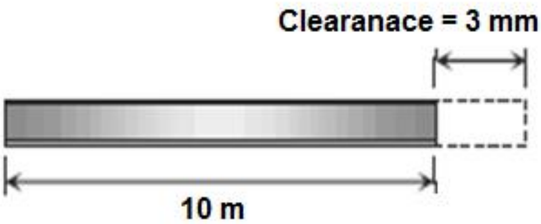
a) At what temperature will the rails just touch each other?

b) What stress would be induced in the rails at that temperature if there were no initial clearance?



Solution:

a)



Temperature at which δ_T = 3 mm:

$$\delta_T = \alpha L(\Delta T)$$
$$\delta_T = \alpha L(T_f - T_i)$$
$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$
$$T_f = 40.64°\text{C}$$

b)

Required stress:

$$\delta = \delta_T$$
$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T)$$
$$\sigma = \alpha E(T_f - T_i)$$
$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$
$$\sigma = 60 \text{ MPa}$$